

You might find the following expressions for exponential functions useful on the exam

$$f(t) = a(b)^t$$

$$P(t) = a \left(1 + \frac{r}{n}\right)^{nt}$$

$$P(t) = a(1+r)^t$$

$$h(t) = ae^{kt}$$

★
Make sure you know how to do this

1. Consider the following table for a function $h(x)$:

x	-1	0	1	2	3
$h(x)$	81	135	225	375	625

$$f(t) = ab^t$$

$$f(t) = 135(1.666)^t$$

(a) (5 points) Does the table represent an exponential function? If so, give a formula for the function. If not, explain your answer in a sentence or two.

$$b = \frac{ab^0}{ab^{-1}} = \frac{f(0)}{f(-1)} = \frac{135}{81} \approx 1.6666$$

$$b = \frac{ab^1}{ab^0} = \frac{f(1)}{f(0)} = \frac{225}{135} \approx 1.6666$$

$$b = \frac{ab^2}{ab^1} = \frac{f(2)}{f(1)} = \frac{375}{225} = 1.6666$$

$$b = \frac{ab^3}{ab^2} = \frac{f(3)}{f(2)} = \frac{625}{375} = 1.6666$$

Expon

2. Suppose that \$5,000.00 is invested in an account and after 1 year there is \$5,635.00 in the account. For all parts of this problem, be sure to **show all work** to receive credit.

(a) (3 points) By what percentage is the investment growing every year? That is, find the annual growth rate as percentage.

$$\frac{\text{new} - \text{old}}{\text{old}} \times 100$$

$$\frac{5,635 - 5,000}{5,000} \times 100$$

← To turn to percent

12.7 %

(b) (3 points) Write an equation $Q(t)$ for the amount of money in the account after t years.

~~$Q(t) = 5000(1 + 0.127)^t$~~
 ~~$Q(t) = 5000(1.127)^t$~~
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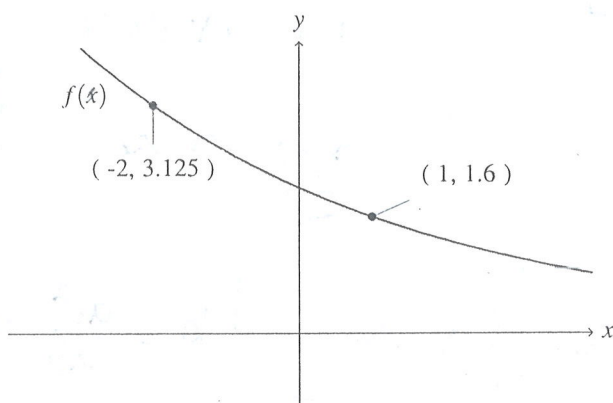
growth factor

$$Q(t) = 5000(1.127)^t$$

(c) (3 points) How much money will be in the account after 5 years?

$$5000(1.127)^5$$

3. Below is a graph of an exponential function $f(x)$. For all parts of this problem, be sure to **show all work** to receive credit.



(a) (5 points) Find an equation for $f(x)$ in the form $f(x) = ab^x$.

① Compare : $b^3 = \frac{ab}{ab^{-2}} = \frac{f(1)}{f(-2)} = \frac{1.6}{3.125} = 0.512$ ← (cube root to find b)
 $b = 0.8$ ← = $(0.512)^{\wedge}(1/3) = \sqrt[3]{0.512}$

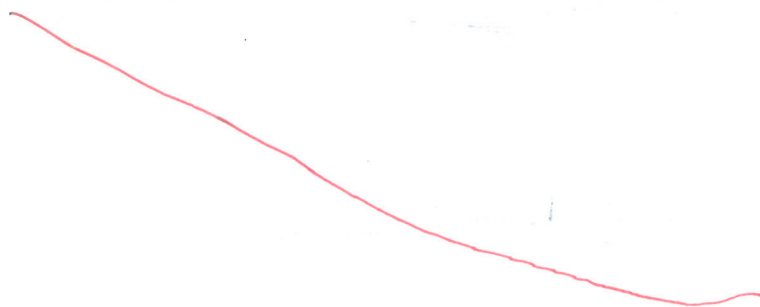
② Find a : $f(x) = ab^x = a(0.8)^x$

$1.6 = f(1) = a(0.8)^1 = 0.8 \cdot a$

$2 = a$

$f(x) = 2(0.8)^x$

(b) (4 points) Express $f(x)$ as an equation in the form $f(x) = ae^{kx}$.



4. A cup of coffee is left sitting on a table and the temperature (in degrees Fahrenheit) of the coffee is given by

$$F(t) = 90(0.95)^t + 65,$$

where t is the number of minutes since the coffee was left on the table. For all parts of this problem, be sure to **show all work** to receive credit.

- (a) (3 points) What temperature was the cup of coffee when it was left on the table?

- (b) (3 points) What temperature will the coffee be in 10 minutes?

- (c) (3 points) What is the average rate that the coffee is cooling between $t = 0$ minutes and $t = 10$ minutes? Be sure to **show all work** to receive credit, and be sure to **include units** on your **final answer**.

- (d) (2 points) How many minutes are needed before the coffee reaches 136 degrees?

Initials: _____

5. A certificate of deposit (CD) is a type of investment. Suppose that a particular CD offers a nominal rate of 4% per year, compounded monthly, and suppose that \$15,000 is invested into the CD account. For all parts of this problem, be sure to **show all work** to receive credit.

(a) (3 points) Give an equation, $P(t)$ for the balance of the account after t years.

$$P(t) = P \left(1 + \frac{r}{n} \right)^{nt}$$
$$P(t) = 15000 \left(1 + \frac{0.04}{12} \right)^{12t} \quad \star$$

(b) (3 points) What is the effective annual rate for this account? Give your **final answer** as a percent, accurate to **at least two decimal places** if you round.

$$\text{e.a.i.r} = \left(1 + \frac{r}{n} \right)^n - 1$$
$$= \left(1 + \frac{0.04}{12} \right)^{12} - 1 \quad \star$$

To put in percent multiply by 100

$$= \left[\left(1 + \frac{0.04}{12} \right)^{12} - 1 \right] \times 100 \%$$

(c) (3 points) CDs require you to hold your money in the account for a set amount of time until it "matures" and you can take your balance out. If the CD in our example matures in 24 months, how much money will be in the account when the CD matures?

t is in years!

$$P(2) = 15000 \left(1 + \frac{0.04}{12} \right)^{12 \cdot 2}$$

6. Each part of this problem is unrelated. For all parts of this problem, be sure to **show all work** to receive credit.

(a) (2 points) Let $y = f(x) = 4x^3 - 12$. Find $f^{-1}(y)$.

$y = 4x^3 - 12$
 ① ~~Solve~~ Solve for x
 $y + 12 = 4x^3 \rightarrow \frac{y + 12}{4} = x^3 \rightarrow \sqrt[3]{\frac{y + 12}{4}} = x$

(b) (1 point) Rewrite $\log_4(1024) = 5$ as an exponential equation.

(c) (1 point) Rewrite $e^{1.25276} = 3.5$ as a logarithmic equation.

(d) (1 point) Evaluate $\log_3(81)$.

(e) (3 points) Rewrite $\ln(17e^2)$ as the sum of two logarithms, then simplify your answer using logarithm rules. Be sure to **show all work** so that your simplification steps are clear.

Initials: _____

7. Technetium-99m (Tc-99 here) is a radioactive isotope that is a byproduct of some fission reactors and is commonly used in medical imaging. For a certain piece of Tc-99 (initially weighing 5 grams), the remaining amount (in grams) of Tc-99 after t hours is given by

$$M(t) = 5e^{-0.1155t}$$

For all parts of this problem, be sure to **show all work** to receive credit.

- (a) (3 points) How much Tc-99 is left after 8 hours?

$$M(8) = 5e^{-0.1155(8)}$$

decay

- b) What is the effective growth rate?

$$\boxed{e^{-0.1155} - 1}$$

- (b) (3 points) What is the half-life of Tc-99?

- (c) (3 points) Using logarithms, determine how many hours it will take for there to be only one gram of Tc-99 left.